Mathematics Methods Notes

Differentiation

Trigonometric functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

Product rule

Form of $a\ (sin/cos/tan)\ x$

Example 1:
Differentiate $y = 15 \sin x$.

Let $u = 15$ $\quad v = \sin x$

$\frac{du}{dx} = 0$ $\quad \frac{dv}{dx} = \cos x$

$\frac{d}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= 15 \cos x + \sin x\, 0$
$= 15 \cos x$

Example 2:
Differentiate $y = 2x \tan x$.

Let $u = 2x$ $\quad v = \tan x$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = \sec^2 x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= 2x \sec^2 x + 2 \tan x$

Form of $a^n\ (sin/cos/tan)\ x$

Example:
Differentiate $y = x^3 \cos x$.

Let $u = x^3$ $\quad v = \cos x$

$\frac{du}{dx} = 3x^2$ $\quad \frac{dv}{dx} = -\sin x$

$\frac{d}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= x^3(-\sin x) + \cos x(3x^2)$
$= -x^3 \sin x + 3x^2 \cos x$

Chain rule

Form of $(sin/cos/tan)ax$

Example:
Differentiate $y = \sin 3x$.

Let $u = 3x$ $\quad y = \sin u$

$\frac{du}{dx} = 3$ $\quad \frac{dy}{du} = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= 3\cos u$
$= 3 \cos 3x$

Form of $sin^n / cos^n / tan^n\ x$

Example 2:
Differentiate $y = \sin^2 x$.

Let $u = \sin x$ $\quad y = u^2$

$\frac{du}{dx} = \cos x$ $\quad \frac{dy}{du} = 2u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= 2u \cos x$
$= 2\sin x \cos x$
$= \sin 2x$

Exponential function

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

$$\frac{d}{dx}e^{f(x)} = f'(x) \cdot e^{f(x)}$$

Product rule

Form of $f(x) \cdot e^x$

Example 1:
Differentiate $y = \sqrt[3]{x}\, e^x$.

Let $u = e^x$ $\quad v = x^{\frac{1}{3}}$

$\frac{du}{dx} = e^x$ $\quad \frac{dv}{dx} = \frac{1}{3\sqrt[3]{x^2}}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= \frac{e^x}{3\sqrt[3]{x^2}} + x^{\frac{1}{3}} e^x$

Example 2:
Differentiate $y = \cos^2 x\, e^x$.

Let $u = e^x$ $\quad v = \cos^2 x$

$\frac{du}{dx} = e^x$ $\quad \frac{dv}{dx} = -2\sin x \cos x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= e^x(-2 \sin x \cos x) + e^x \cos^2 x$
$= -2\, e^x \sin x \cos x + e^x \cos^2 x$

Chain rule

Form of ae^{bx}

Example 1:
Differentiate $y = 5e^{-8x}$.

Let $u = -8x$ $\quad y = 5e^u$

$\frac{du}{dx} = -8$ $\quad \frac{dy}{du} = 5e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= 5e^u \times -8$
$= -40e^{-8x}$

Example 2:
Differentiate $y = e^{4x}$.

Let $u = 4x$ $\quad y = e^u$

$\frac{du}{dx} = 4$ $\quad \frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= 4 \times e^u$
$= 4e^{4x}$

Form of $ae^{f(x)}$

Example:
Differentiate $y = e^{\cos x}$.

Let $u = \cos x$ $\quad y = e^u$

$\frac{du}{dx} = -\sin x$ $\quad \frac{dy}{du} = e^u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= -\sin x \times e^u$
$= -\sin x\, e^{\cos x}$

Logarithmic function

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

Product rule

Form of $a\ \ln x$

Example 1:
Differentiate $y = (x + 1) \ln x$.

Let $u = (x + 1)$ $\quad v = \ln x$

$\frac{du}{dx} = 1$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= \frac{x+1}{x} + \ln x$

Example 2:
Differentiate $y = 2e^{\frac{x}{3}} \ln x$.

Let $u = 2e^{\frac{x}{3}}$ $\quad v = \ln x$

$\frac{du}{dx} = \frac{2e^{\frac{x}{3}}}{3}$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= \frac{2e^{\frac{x}{3}}}{x} + \frac{\ln x\ 2e^{\frac{x}{3}}}{3}$

Example 3:
Differentiate $y = 3 \ln x$.

Let $u = 3$ $\quad v = \ln x$

$\frac{du}{dx} = 0$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$= \frac{3}{x}$

Chain rule

Form of $\ln(ax + b)$

Example 1:
Differentiate $y = \ln(e^{2x} + 2)$.

Let $u = e^{2x} + 2$ $\quad y = \ln u$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dy}{du} = \frac{1}{u}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= \frac{2e^{2x}}{e^{2x}+2}$

Example 2:
Differentiate $y = \ln(x + 2x^2)$.

Let $u = x + 2x^2$ $\quad y = \ln u$

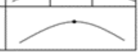
$\frac{du}{dx} = 1 + 4x$ $\quad \frac{dy}{du} = \frac{1}{u}$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$= \frac{1 + 4x}{x + 2x^2}$

Max. point

Sign test

	x^-	x_0	x^+
$\frac{dy}{dx}$	$+ve$	0	$-ve$
Slope	/	—	\
Stationary point			



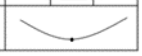
2^{nd} derivative test,

$$\frac{d^2y}{dx^2} < 0$$

Min. point

Sign test

	x^-	x_0	x^+
$\frac{dy}{dx}$	$-ve$	0	$+ve$
Slope	\	—	/
Stationary point			

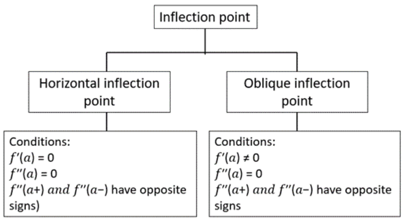


2^{nd} derivative test,

$$\frac{d^2y}{dx^2} > 0$$

Inflection point

$$f''(x) = 0 \quad / \quad \frac{d^2y}{dx^2} = 0$$



Small change and rate of change

Small change,

$$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$$

Approximation,

$$Y_{new} = Y_{old} + \delta y$$

Example:
Determine the approximate percentage change of $y = 3x^2$ when x increases by 1%.

$\delta x = 1.01x - 1x$
$= 0.01x$

$y = 3x^2$
$\frac{dy}{dx} = 6x$

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$
$\delta y = 6x \times 0.01x$
$= 0.06x^2$

$\frac{\delta y}{y} \times 100$
$= \frac{0.06x^2}{3x^2} \times 100$
$= 2\%$

Example 2:
A drop of methyl blue solution is dripped into a bowl of water and small circle ripples are formed continuously. Given that the radius of ripple increases at rate of $0.5\ cms^{-1}$, find the rate of change of the area of ripple in terms of π when the diameter is 0.4 cm.

$A = \pi r^2$
$\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$
$= 2\pi r(0.5)$
$= 2\pi(0.4 \div 2)(0.5)$
$= 0.2\pi\ cm^2 s^{-1}$

Integration

Linear

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + c$$

Trigonometric

$$\int cos\ x\ dx = sin\ x + c$$

$$\int cos\ (ax+b)\ dx = \frac{1}{a} sin\ (ax+b) + c$$

$$\int sin\ x\ dx = -cos\ x + c$$

$$\int sin\ (ax+b)\ dx = -\frac{1}{a} cos\ (ax+b) + c$$

$$\int sec^2 x\ dx = tan\ x + c$$

Logarithmic

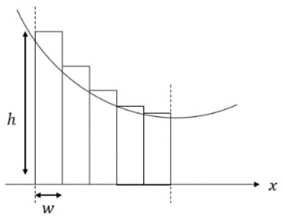
$$\int \frac{1}{x}\ dx = \ln x + c$$

$$\int \frac{1}{ax+b}\ dx = \ln(ax+b) + c$$

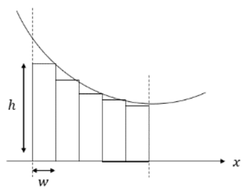
$$\int \frac{f'(x)}{f(x)}\ dx = \ln(f(x)) + c$$

Estimates of graph

Given a curve with function $f(x)$



Area circumscribed



Area circumscribed

To find each area of strips (rectangles):

$$A = f(x) / h \times w$$

Estimating the area of curve,

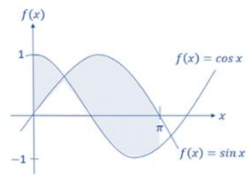
$$Area \approx \frac{A_{circumscribed} + A_{inscribed}}{2}$$

Tips for finding area bounded by two functions:

Use the function above minus the function below.

Example:

Find the area trapped between $f(x) = sin\ x$ and $f(x) = cos\ x$ for the range $0 \le x \le \pi$.



Graph intersects at $x = \frac{\pi}{4}$ in the range $0 \le x \le \pi$

$$A = \int_0^{\frac{\pi}{4}} cos\ x - sin x\ dx + \int_{\frac{\pi}{4}}^{\pi} sin\ x - cos x\ dx$$

$$= 2\sqrt{2}\ units^2$$

Fundamental theorem

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

Example:

Use the fundamental theorem of calculus to evaluate $\int_0^1 x^2 + e^x\ dx$. Give your answer in terms of e.

$$\int_0^1 x^2 + e^x\ dx = [\frac{x^3}{3} + e^x]_0^1$$

$$= [\frac{1}{3} + e] - [\frac{0}{3} + e^0]$$

$$= -\frac{2}{3} + e$$

$$\frac{d}{dx}[\int_a^x f(t)\ dt = f(x)$$

Example:

Determine $\frac{d}{dx}[\int_1^x t^2 + 2]\ dt$.

$$\frac{d}{dx}[\int_1^x t^2 + 2]\ dt = x^2 + 2$$

$$\frac{d}{dx}[\int_a^{g(x)} f(t)]\ dt = f[g(x)] \times g'(x)$$

Example:

Find $\frac{d}{dx}[\int_1^{x+1} t]\ dt$.

$$\frac{d}{dx}[\int_1^{x+1} t]\ dt = (x+1) \times \frac{d}{dx}(x+1)$$

$$= x + 1$$

Using fundamental theorem of calculus with two variable limits of integration

Example:

Find $f'(t)$ of $f(t) = \int_t^{3t} x^3\ dx$.

$$\frac{d}{dt}\int_t^{3t} x^3\ dx = \frac{d}{dt}\int_0^{3t} x^3\ dx + \frac{d}{dt}\int_t^0 x^3\ dx$$

$$= (3t)^3 \times \frac{d}{dt}(3t) - \frac{d}{dt}\int_0^t x^3\ dx$$

$$= 3(27t^3) - t^3$$

$$= 80t^3$$

$$\int_b^a \frac{d}{dt}[f(t)]\ dt = f(a) - f(b)$$

Example:

Find $\int_\pi^{x^2} \frac{d}{dt}(2t^2 + t)dt$.

$$\int_\pi^{x^2} \frac{d}{dt}(2t^2 + t)dt = [2(x^2)^2 + x^2] - [2\pi^2 + \pi]$$

$$= 2x^4 + x^2 - 2\pi^2 - \pi$$

Additivity and linearity of definite integrals

$$\int_a^a f(x)dx = 0$$

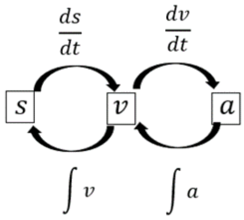
$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

$$\int_a^b k \times f(x)dx = k\int_a^b f(x)dx$$

$$\int_a^b [f(x) \pm g(x)]\ dx = \int_a^b f(x)\ dx \pm \int_a^b g(x)\ dx$$

Rectilinear motion



Displacement

Example:

A particle accelerates at $5\ ms^{-2}$ which is constant along the s-axis. It has a velocity of $-1\ ms^{-2}$ at the start of the journey. Find the displacement of the particle within the range of time, $2 \leqslant t \leqslant 4$.

$$v(t) = -1 + 5t$$

$$S = \int_2^4 v\ dt$$

$$= \int_2^4 -1 + 5t\ dt$$

$$= [-t + \frac{5t^2}{2}]_2^4$$

$$= 28\ m$$

Distance

Example:

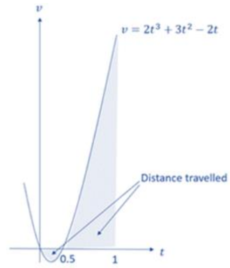
The velocity of a moving particle is equated by $v = 2t^3 + 3t^2 - 2t$. Find the total distance travelled by particle from its origin point till $t = 1s$.

$$S = \int_0^1 |v|\ dt$$

$$= \int_0^{0.5} |2t^3 + 3t^2 - 2t|\ dt + \int_{0.5}^1 2t^3 + 3t^2 - 2t\ dt$$

$$= 0.09375 + 0.59375$$

$$= 0.6875\ m$$



Velocity

Example:

The velocity function of a moving particle is given by $v(t) = e^{sin\ 2t}$, find the velocity of the particle at $t = 2\ s$. Do not evaluate your answer.

$$v(t) = e^{sin\ 2t}$$

$$v(2) = e^{sin\ 2(2)}$$

$$= e^{sin\ 4}\ ms^{-1}$$

Acceleration

Example:

A moving particle travells along a straight path and passes through a fixed point. Its velocity, $v\ ms^{-1}$ is given by $v = 3t^2 + 6t$. Determine the acceleration of the particle when $t = 2$.

$$v = 3t^2 + 6t$$

$$a = \frac{dv}{dt}$$

$$a(t) = 6t + 6$$

$$a(2) = 6(2) + 6$$

$$= 18\ ms^{-2}$$

Exact values of trigo. ratio

	0	$30/\frac{\pi}{6}$	$45/\frac{\pi}{4}$	$60/\frac{\pi}{3}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

	$90/\frac{\pi}{2}$	$180/\pi$	$270/\frac{3\pi}{2}$	$360/2\pi$
sin	1	0	−1	0
cos	0	−1	0	1
tan	–	0	–	0

Trigonometric identities

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \sin(B)\cos(A)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

Economic application of differentiation

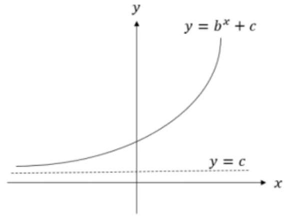
Application	Function
Production cost	$C(x)$
Revenue	$R(x)$
Profit	$P(x) = C(x) - R(x)$
Break even	$C(x) = R(x)$
Average cost	$\frac{c(x)}{x}$
Marginal Profit	$P'(x)$
Marginal Revenue	$R'(x)$
Marginal cost	$C'(x)$
Approximate cost of producing one more unit after x th unit has been produced and sold	$C''(x)$
Maximum profit	$C'(x) = R'(x)$ or $P'(x)$

Exponential function

Characteristics of graphs of exponential function:

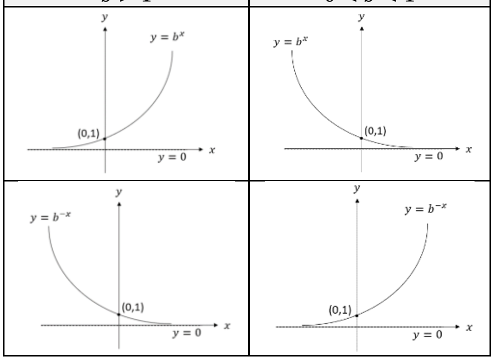
- Graph is asymptotic to the x-axis as x approaches infinity

  Horizontal asymptote is $y = c$ (general graph horizontal asymptote is $y = 0$)



- Domain are real numbers
- Graph is smooth and continuous
- y-intercept at $(0, a + c)$ (general graph y-intercept at (0,1))

$b > 1$	$0 < b < 1$



Euler's number

Expression	Euler's number expression
$(1 + \frac{k}{n})^n$	e^k
$(1 + \frac{1}{kn})^n$	$e^{\frac{1}{k}}$

Example 1:

Given that $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$, evaluate $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^{2n}$.

$$\lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^{2n} = e^2$$

Example 2:

Given that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ Show that $e \approx 2.7182$ by substituting $x = 1$.

Using first 7 terms,
Sub $x = 1$,

$$e^1 = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!}$$
$$e = 2.71825$$
$$\approx 2.7182$$

Continuous exponential growth/ decay

$$A = A_o e^{kt}$$

Example 1:
After the re-snap, half of the population of humans revives continuously such that $\frac{dp}{dt} = 5.2\,t$ in which t is the population of humans after t hours. How long does it take for the population of human to be 7.7 billion? Give your answer in 3 significant figures

$$7.7 = (7.7 \div 2)e^{5.2\,t}$$
$$7.7 = 3.85e^{5.2t}$$
$$e^{5.2t} = 2$$
$$5.2t = \frac{\log_{10} 2}{\log_{10} e}$$
$$5.2t = 0.69314718$$
$$t = 0.133\ hours$$

Example 2:
The population of uncontrolled rats is initially 400. The estimated growth rate of rates after n, months is given by $400e^{0.12n}$. Estimate the population of rats after 2 months.

$$P = 400e^{0.12n}$$
$$= 400e^{0.12(2)}$$
$$= 508.5 \approx 509$$

Example 3:
The initial population of bees of 5000 is decreasing at rate of $\frac{dy}{dx} = -0.9\,y$ in which y is the population of bees while x is the years from initial population. Find the population of bees after 5 years.

$$P = P_o e^{kx}$$
$$= 5000e^{-0.9(5)}$$
$$= 55.545 \approx 56$$

Random variable

Discrete random variable

A variable whose values (countable values) are obtained by counting

Probability distribution properties (characteristics):

- Probabilities for each value of X lies in the interval of $0 \le P(X = x) \le 1$
- Sum (total) of the probabilities is 1

(A) Probability of a discrete random variable by choosing with replacement

$$P(X = x) = {}^nC_r(p)^r(1-p)^{n-r}$$

Example:
A bag of chips contains 4 red and 2 blue chips. Three chips are drawn randomly without replacement. Draw a probability distribution for X: number of blue chips drawn.

$$P(red) = \frac{4}{6} = \frac{2}{3} \qquad P(blue) = \frac{2}{6} = \frac{1}{3}$$

$f(x) = {}^3C_x(\frac{1}{3})^x(\frac{2}{3})^{3-x}$ for $x =$ 0,1,2, ...

X	0	1	2
$P(X = x)$	$\frac{8}{27}$	$\frac{4}{9}$	$\frac{2}{9}$

(B) Probability of a discrete random variable by choosing without replacement

$$P(X = x) = \frac{{}^{n_1}C_r\,{}^{n_2}C_{n-r}}{{}^{n_1+n_2}C_n}$$

Example:
A bag of marbles contains 4 yellow-green and 3 blue-magenta marbles. Three marbles are drawn randomly without replacement. Draw a probability distribution for X: number of yellow-green marble.

$$P(yellow-green) = \frac{4}{7} \qquad P(blue-magenta) = \frac{3}{7}$$

$$f(x) = \frac{{}^4C_x\,{}^3C_{3-x}}{{}^{4+3}C_3}$$

X	0	1	2	3
$P(X = x)$	$\frac{1}{35}$	$\frac{12}{35}$	$\frac{18}{35}$	$\frac{4}{35}$

Mean & variance of DRV

$$E(X) = \sum x\,P(X = x)$$
$$Var(X) = \sum (x-\mu)^2 \times P(X = x)$$

Or

$$Var(X) = E(X^2) - [E(X)]^2$$

Continuous random variable

A variable which takes any values over intervals and whose values (measurable values) are obtained by measuring.

Probability density function (characteristics):

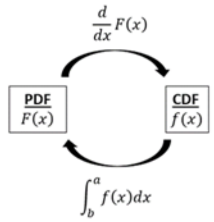
- Sum (total) of the probabilities is 1: $\int_b^a f(x)\,dx = 1$
- $f(x) \ge 0$ for interval $a \le x \le b$
- $P(X = k) = 0$ which is same as $\int_k^k f(x)\,dx = 0$
- $P(X \le k) = P(X < k) + P(X = k) = P(X < k)$

Finding probability density function given a cumulative distribution function

$$f(x) = \frac{d}{dx}F(x)$$

CDF

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$



Example:
The probability density function f of a continuous random variable T is given by,

$$f(t) = \begin{cases} \frac{1}{24}t & 0 \le t \le 4 \\ \frac{1}{4} - \frac{1}{48}t & 4 \le t \le 12 \\ 0 & \text{Otherwise} \end{cases}$$

- Find the cumulative distribution function for T.

$$F(t) = \begin{cases} 0 & t < 0 \\ \int \frac{1}{24}t\,dt = \frac{t^2}{48} & 0 \le t \le 4 \\ \int_4^t \frac{1}{4} - \frac{1}{48}t\,dt + \int_0^4 \frac{t}{24}dt = \frac{t}{4} - \frac{t^2}{96} - \frac{1}{2} & 4 \le t \le 12 \\ 1 & t > 12 \end{cases}$$

- Find $P(3 < T < 12)$.

$$P(3 < T < 12) = F(12) - F(3)$$
$$= \frac{12}{4} - \frac{(12)^2}{96} - \frac{1}{2} - \frac{3^2}{48}$$
$$= \frac{13}{16}$$

or

$$P(3 < T < 12) = \int_4^{12} \frac{1}{4} - \frac{1}{48}t\,dt + \int_3^4 \frac{t}{24}dt$$
$$= \frac{13}{16}$$

Uniform variable

Discrete uniform variable

Discrete uniform variable properties (characteristics):

- n values in the range has equal probability $\frac{1}{n}$ (the probability of uniformly spaced possible values is equal)

Probability mass function:

$P(X = x) = \frac{1}{n}$ for $x = 1,2,3,4 \ldots, n$

Mean & variance

$$E(X) = \frac{n+1}{2}$$

$$Var(X) = \frac{n^2 - 1}{12}$$

Probability density function:

$$f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

Notation:

$X \sim U(a, b)$ if $a < x < b$

$X \sim U[a, b]$ if $a \le x \le b$

Mean & variance

$$E(X) = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

Cumulative density function

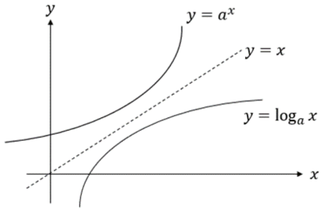
$$P(X \le x) = \int_a^x \left(\frac{1}{b-a}\right) dx$$

$$= [\frac{x}{b-a}]_a^x$$

$$= \frac{x}{b-a} - \frac{a}{b-a}$$

$$= \frac{x-a}{b-a}$$

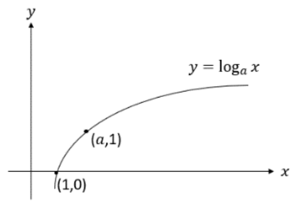
Logarithm



Condition

- $a > 0,\ a \ne 1$
- x cannot be negative

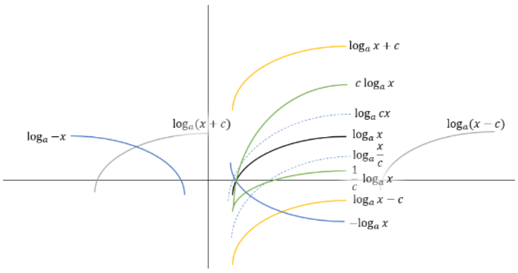
General graph of logarithmic function



Condition: $a > 1$

Main features/ characteristics

- Graph is continuous
- Domain is for $x > 0$
- Range is for all real numbers
- Vertical asymptote $x = 0$
- x-intercept at (1,0)

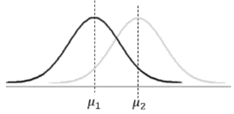
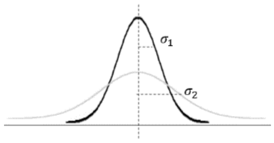


Transformation	Effect
$\log_a x + c$	Graph shifts upwards by c units
$\log_a x - c$	Graph shifts downwards by c units
$\log_a(x + c)$	Graph shifts leftwards by c units
$\log_a(x - c)$	Graph shifts rightwards by c units
$\log_a cx$ $c > 1$	Graph compress/ shrunk horizontally by a factor of c
$\log_a \frac{x}{c}$ $c > 1$	Graph stretch horizontally by a factor of c
$c \log_a x$ $c > 1$	Graph stretch vertically by a factor of c
$\frac{1}{c} \log_a x$ $c > 1$	Graph compress/ shrunk vertically by a factor of c
$-\log_a x$	Graph reflects about $x = 0$/ x –axis
$\log_a -x$	Graph reflects about $y = 0$/ y-axis

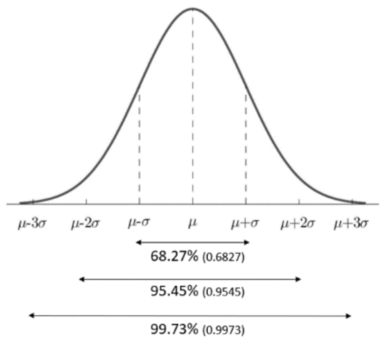
Normal distribution

Basic properties of normal distribution:

- It is symmetric about the mean
- The mean = the mode = the median
- The curve is unimodal (one peak), maximum point at $(\mu, \frac{1}{\sigma\sqrt{2\pi}})$
- The curve approaches but never touches, the x-axis, as it extends farther and farther away from the mean $(-\infty < x < \infty)$
- Total area under the curve = 1 $\left(\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = 1\right)$
- Mean, μ and standard deviation, σ

Mean, μ	The larger the mean, the greater the shift towards the right  $\mu_2 > \mu_1$
Standard deviation, σ	The larger the standard deviation, the greater the vertical compression  $\sigma_2 > \sigma_1$

Area under normal distribution and its corresponding standard deviation away from mean, μ.



Example:

A random variable T has a normal distribution with mean of 39 and variance, σ^2. Given that $P(X > 42.5) = 0.098876$, find the standard deviation.

$P(X > 42.5) = 0.098876$

$P(Z > \frac{42.5 - 39}{\sigma}) = 0.098876$

Invnorm (1 – 0.098876, 0,1)

$\frac{42.5 - 39}{\sigma} = 1.28798$

$3.5 = 1.28798\sigma$

$\sigma = 2.71743$

Sampling

Ways to sample randomly:

- Use random number generator (without repetition of any numbers) to generate random integer
- Use lottery method by writing numbers (eg: 1 to 10) on individual papers and draw the numbers randomly.

Selection Bias: Issues with sampling

- **Under coverage:** when members of the population aren't adequately represented.
- **Nonresponse:** views of non-respondents are missed as they are unwilling and/or unable to participate in the survey.
- **Voluntary Response:** sampling people who will only willingly participate.

Population & sample proportion

Population proportion	$p = \frac{X}{N}$
Sample proportion	$\hat{p} = \frac{x}{n}$

Conditions for sampling distribution to be approximately large:

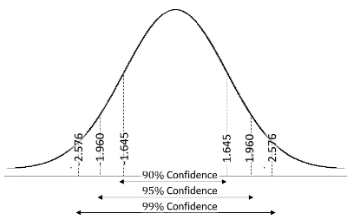
- $n > 30$
- $np > 5$
- $nq > 5$

(the conditions stated above varies and are not definite)

Standard score

$$Z = \frac{X - \mu}{\sigma} = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Confidence interval



Percentage %	No. of standard deviation/ z –score
90%	1.645
95%	1.960
99%	2.576

**take z-score to 3 decimal places*

Formula:

$$\hat{p} \pm z\sigma$$

Represented as:

___% confidence interval $(\hat{p} - z\sigma, \hat{p} + z\sigma)$

Interpretation

We could expect that ___% of the ___% confidence interval contains the true/population proportion

Example:

90% confidence level for 10 samples.